## GCE Examinations

## Advanced Subsidiary

## Core Mathematics C1

Paper B<br>Time: 1 hour 30 minutes

## Instructions and Information

Candidates may NOT use a calculator in this paper
Full marks may be obtained for answers to ALL questions.
Mathematical formulae and statistical tables are available.
This paper has ten questions.

## Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.
1.

$$
f(x)=(\sqrt{x}+3)^{2}+(1-3 \sqrt{x})^{2}
$$

Show that $\mathrm{f}(x)$ can be written in the form $a x+b$ where $a$ and $b$ are integers to be found.
2. The curve $C$ has the equation

$$
y=x^{2}+a x+b
$$

where $a$ and $b$ are constants.
Given that the minimum point of $C$ has coordinates $(-2,5)$, find the values of $a$ and $b$.
3. The sequence $u_{1}, u_{2}, u_{3}, \ldots$ is defined by

$$
u_{n}=2^{n}+k n,
$$

where $k$ is a constant.
Given that $u_{1}=u_{3}$,
(a) find the value of $k$,
(b) find the value of $u_{5}$.
4. Given that

$$
\begin{equation*}
\frac{\mathrm{d} y}{\mathrm{~d} x}=2 x^{3}+1 \tag{6}
\end{equation*}
$$

and that $y=3$ when $x=0$, find the value of $y$ when $x=2$.
5. $\mathrm{f}(x)=4 x-3 x^{2}-x^{3}$.
(a) Fully factorise $4 x-3 x^{2}-x^{3}$.
(b) Sketch the curve $y=\mathrm{f}(x)$, showing the coordinates of any points of intersection with the coordinate axes.
6. The straight line $l$ has the equation $x-2 y=12$ and meets the coordinate axes at the points $A$ and $B$.

Find the distance of the mid-point of $A B$ from the origin, giving your answer in the form $k \sqrt{5}$.
7. (a) Given that $y=2^{x}$, find expressions in terms of $y$ for
(i) $2^{x+2}$,
(ii) $2^{3-x}$.
(4)
(b) Show that using the substitution $y=2^{x}$, the equation

$$
2^{x+2}+2^{3-x}=33
$$

can be rewritten as

$$
\begin{equation*}
4 y^{2}-33 y+8=0 . \tag{2}
\end{equation*}
$$

(c) Hence solve the equation

$$
\begin{equation*}
2^{x+2}+2^{3-x}=33 . \tag{4}
\end{equation*}
$$

8. Given that

$$
y=2 x^{\frac{3}{2}}-1,
$$

(a) find $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$,
(b) show that

$$
\begin{equation*}
4 x^{2} \frac{\mathrm{~d}^{2} y}{\mathrm{~d} x^{2}}-3 y=k \tag{2}
\end{equation*}
$$

where $k$ is an integer to be found,
(c) find

$$
\begin{equation*}
\int y^{2} \mathrm{~d} x \tag{6}
\end{equation*}
$$

9. The second and fifth terms of an arithmetic series are 26 and 41 repectively.
(a) Show that the common difference of the series is 5 .
(b) Find the 12th term of the series.

Another arithmetic series has first term -12 and common difference 7 .

Given that the sums of the first $n$ terms of these two series are equal,
(c) find the value of $n$.
10.


Figure 1
Figure 1 shows the curve $y=x^{2}-3 x+5$ and the straight line $y=2 x+1$. The curve and line intersect at the points $P$ and $Q$.
(a) Using algebra, show that $P$ has coordinates $(1,3)$ and find the coordinates of $Q$.
(b) Find an equation for the tangent to the curve at $P$.
(c) Show that the tangent to the curve at $Q$ has the equation $y=5 x-11$.
(d) Find the coordinates of the point where the tangent to the curve at $P$ intersects the tangent to the curve at $Q$.

